SEM-4 (MATHEMATICS GENERAL)
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BMG4CC1D TOPIC - SUBGROUPS

Subgroup: If we consider two groups, one composed of a set S and the operation $*$ and the other composed of a non-empty ~~sub~~set H and the same operation $*$, then we say that the later is a subgroup of the former group, if $H \subset S$.

Examples of a subgroup

1) The multiplicative group $\{-1, 1\}$ is a proper subgroup of the multiplicative group $\{-1, 1, i, -i\}$.

2) The additive group of rational numbers is a subgroup of real numbers.

3) The multiplicative group of positive real numbers is a sub-group of the multiplicative group of all non-zero real numbers.

4) The multiplicative group of non-zero rationals is a subgroup of non-zero reals.

Theorem 1 The identity of a subgroup is the same as that of the group.

Proof: Let H be the subgroup of the group G for certain operation and let e and e' be the identity elements of G and H respectively.

Now if $a \in H$, then $a \in G$ and $ae = a$, since e is the identity element of G. Also $a \in H$

Therefore $ae' = a$, since e' is the identity element of H.

Thus $ae = ae'$, which gives $e = e'$

follows

Theorem 2 The inverse of any element of a sub-group is the same as the inverse of the same element regarded as an element of the group.

Proof: Let H be the subgroup of the group G for certain operation. and e be the common identity element.

If $a \in H$ then $a \in G$. ~~and~~

Let b be the inverse of $a \in H$ and c be the inverse of $a \in G$

Then $ab = e$, since b is the inverse of $a \in H$.

and $ac = e$, since c is the inverse of $a \in G$

Therefore $ab = ac$ which gives $b = c$.

Hence the theorem follows.